VOLUME OF A PARALLELEPIPED

The volume of the parallelepiped shown in the figure is given by

$$V = \{\text{base area}\} \times \{\text{height}\},$$

where the base area is $AB\sin\theta$ or equivalently $|\,\vec{\mathbf{A}} \times \vec{\mathbf{B}}\,|$.

As should be clear from the figure, the height H is the length (magnitude) of the projection of vector $\vec{\mathbf{C}}$ along the direction of $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$, i.e.,

$$H = \vec{\mathbf{C}} \cdot \hat{\mathbf{u}},$$

where

$$\hat{\mathbf{u}} = \frac{\vec{\mathbf{A}} \times \vec{\mathbf{B}}}{|\,\vec{\mathbf{A}} \times \vec{\mathbf{B}}\,|}$$

Substitution yields

$$\begin{aligned} V &= \left\{|\,\vec{\mathbf{A}} \times \vec{\mathbf{B}}\,|\right\} \times \left\{\vec{\mathbf{C}} \cdot \frac{\vec{\mathbf{A}} \times \vec{\mathbf{B}}}{|\,\vec{\mathbf{A}} \times \vec{\mathbf{B}}\,|}\right\} \\ &= \vec{\mathbf{C}} \cdot (\vec{\mathbf{A}} \times \vec{\mathbf{B}}) \end{aligned}$$